DISCUSSION BEFORE THE RADIO SECTION, 15TH FEBRUARY, 1950.

Dr. R. L. Smith Rose: This paper, which is of unusual interest to so many, presents a problem with which I have been concerned for nearly 25 years. It is a problem which dates almost from the earliest demonstrations that radio waves existed and could be transmitted great distances, over and around the earth's surface, to provide a means of communication.

Yet, in spite of the complications of ionospheric reflection, the curvature of the earth and the obstacles on its surface, the problem remains essentially that of how electromagnetic waves radiate from a vertical aerial over a flat smooth earth. I think it would not be wrong to say that we do not yet fully understand what happens within the first few wavelengths of the aerial, which, despite the increasing use of metre and centimetre wavelengths, may involve distances of 100 miles at the frequencies still in general use for broadcasting. There has, of course, been a good deal of speculation on this point, which, had it provided a solution, would have eased the task of solving that problem which the authors have set. To some extent, the conditions at the boundary between surfaces of changing conductivity reproduce those near the transmitting aerial; the wavefront is set up at the boundary, and we can visualize re-radiation from it. Had this problem been solved, therefore, a solution to the authors' problem might have been reached at an earlier date.

During the past 20 years, various aspects of the problem have come into prominence in connection with direction-finding. It has long been known that, under certain conditions, waves crossing a boundary, such as a coastline, can change in direction, but I am not aware of a satisfactory explanation of that phenomenon.

It is, however, particularly from the interests of the broadcasting authorities that this problem has again been brought to our notice, and the fact that such men as P. P. Eckersley, H. L. Kirke and T. Somerville have been mentioned in the paper indicates the keen interest of the B.B.C. Since the high standards and quality of service which characterize this organization's broadcasting are such as to restrict the radius of the area served

to the length of the ground wave, it is understandable why B.B.C. engineers must be so thoroughly conversant with what happens within the ground-wave conditions.

That, then, is the problem, and it is not a new one. The authors were not satisfied with the various empirical solutions which had been proposed, and, whilst their theoretical solution is not claimed to be a rigid one—by mathematical standards—it was the application of theory which enabled them to produce a theoretically sound quasi-empirical solution from which to organize and carry out, with confidence, the experiments described in the paper. The excellent way in which these experiments were conducted can arouse nothing but admiration. Any worker who has taken radio measurements under field conditions—particularly in ships at sea—will realize the hazards which might arise and the mishaps which may occur. Yet the only unexpected occurrence was that the ship crossed the Channel more quickly than had been anticipated!

It may be said that many of their results could be deduced from certain diagrams of field-strength contours which have been published by earlier workers in this field, but, anticipating such criticism from those who would be wise after the event, the authors state that "though it had been known for a long time that recovery effects can occur in field-strength surveys, it is fair to say that the full effect of the phenomenon at a land/sea boundary had not been realized or its importance appreciated." That sentence, taken in conjunction with the experimental measurements which show a 10-db change in field strength, provides a very valuable appreciation of the whole situation.

In the description of the experiments carried out at a wavelength of 4 m, it is explained that the high-water level was some 3 m below the land. Since these experiments were conducted during the hour preceding, and the hour following, high water, the difference in height between the land and the sea might be assumed as being nearly one wavelength, which seems to call for a correction to the calculated results in regard to height gain. Has that been allowed for in the theoretical curve?

A further point concerns the physical explanation of the occurrences at the boundary. It is known that when waves traverse a horizontal stretch of land the wavefront is tilted forward by a measurable amount but becomes vertical at about 5λ above the earth's surface. Nevertheless, after crossing the land/sea boundary and proceeding some distance out to sea, the entire wavefront is vertical. I do not understand what happens at the boundary. How does this wavefront set itself vertically, and whence arises the resulting increase in field strength? The authors speak of this energy as being fed downwards from the land side to the sea side, but, from the aspect of this tilted wavefront, it appears that the energy need not come from above but could arise by virtue of deflection from below.

Mr. H. L. Kirke: The effect of land-sea paths became a problem in the very early days of planning P. P. Eckersley's regional scheme of broadcasting, and first came into prominence in connection with the Washford Cross station. The next attempt to investigate the mixed path effect was made at the Burghead station in the North of Scotland. An experiment was conducted across the Moray Firth and thence via Helmsdale to Melvick. The results have not been published, as the measurements were not sufficiently accurate because of the mountainous nature of the country and the variability of conductivity. However, we did get a rise of field strength on crossing from land to sea in the reverse direction from a transmitter at Melvick.

Proctor Wilson suggested that in all these cases there should be reciprocity, and an experiment was made in which it was found that if the whole circuit was considered as a 4-terminal network there was reciprocity.

During the early part of the last war, when certain transmissions on medium waves were intended for use overseas, there was some disagreement as to the field strength at a distance over a land-sea-land path. The dispute was whether to use the Eckersley or the Somerville method; the former gave results which were considered to be too optimistic. It was decided to carry out further investigations, the results of which have been published.* These showed that the error with the Somerville method was considerably less than that obtained when using the Eckersley method, but, as the authors point out, the method has no real justification.

I am glad that the authors have found it possible not only to treat this matter theoretically but to carry out these two remarkable and convincing experiments.

I should like to ask the authors whether their method can usefully be applied to very high frequencies radiated over uneven country, which is again a diffraction problem. The problem of calculating field strength over such paths for the estimation of service areas of the proposed v.h.f. transmitters has not been solved, and empirical methods must be used, with, in many cases, surprisingly good results, although never with complete certainty.

The shadow effects of towns is mentioned in the paper. Similar effects have been noticed over London, particularly on 261 m and 203 m. In both cases there is a recovery effect on the far side of London, and the field strength increases.

Height gain has been dealt with in the authors' experiment, and shown to be less after the recovery had taken place than before the boundary. This might well be deduced, because, after the recovery, the field strength is higher, and, as the field strength at a sufficient height is unaffected by the ground, the height gain in the one case must be less than in the other.

Mr. R. A. Rowden: The calculation of field strength over mixed land/sea paths by the authors' method shows very good agreement with measured results obtained by the B.B.C. on broadcast wavelengths. The Somerville method was used by us for many years, and, when restricted to frequencies between 150 and 1 500 kc/s and to cases where the lengths of land and sea path are not very great, e.g. within the service area of a broadcasting station, it gives results of practical value and has the advantage that it can be used with greater speed than the Millington method. On shorter wavelengths and over longer distances, the Somerville method is inaccurate, as shown by the authors' example (Section 6.2). On 300 m, however, over the Slough–Dieppe path, the Somerville method would give a result only 2 db below the Millington value.

The authors used wavelengths of 4 and 100 m to demonstrate the large magnitude of the recovery effect experienced when passing from a land to a sea path. These wavelengths are not very much used for practical communication by the surface wave, and I feel that the authors have tended to over-emphasize the practical importance of the recovery effect by the choice of such short wavelengths.

Mr. H. Stanesby: Three very interesting effects are thrown into relief by the authors' work, and using the methods that have been developed, the magnitude of these effects can for the first time be predicted. These effects, all three of which are well exhibited in Fig. 8 of Part 1, are: (*a*) the recovery of field strength found when, in travelling away from the transmitter, one passes from land to sea, (*b*) the bad effect of a short land path at the end of an oversea path, (*c*) the good effect of a short sea path at the end of an overland path. A very early example of the recovery effect was described by Bown and Gillett in 1924:†

* KIRKE, H. L.: "Calculation of Ground-Wave Field Strength Over a Composite Land and Sea Path," *Proceedings I.R.E.*, 1949, **37**, p. 489.

† BOWN, R., and GILLETT, G. D.: "Distribution of Radio Waves from Broadcasting Stations Over City Districts," *Proceedings of the Institute of Radio Engineers*, 1924 **12**, p. 395.

In plotting the field strength contours of a medium-wave broadcasting station located at the southern end of Manhattan, they found that the field strength fell to about 1 mV/m some five miles north of the transmitter, but rose again to 2·5 mV/m five miles further on in the same direction. Effect (*b*) has been well known for many years and is the reason why Loran stations, and coastal radio stations operating maritime mobile services on medium wavelengths, are placed as near to the edge of the sea as possible. In 1932, Heising* showed that the effect on 5-Mc/s vertically-polarized ground-wave signals of moving a coastal station one mile away from the shore was the same as that caused by reducing the transmitter power in one case to $\frac{1}{6}$ and in another case to $\frac{1}{16}$ of the original value. It seems to be well established that, in passing from sea to land, the field strength falls at least as abruptly as indicated in Fig. 8 of Part 1. Effect (*c*), the beneficial effect of water or high-conductivity earth at the end of a path of lower conductivity, has also been appreciated, but not, perhaps, as fully as it deserves. It emphasizes the advantage, from the aspect of propagation, of siting medium-wave transmitting and receiving stations on mud-banks and salt-marshes, but the civil engineering problems and other practical difficulties cannot be easily overcome.

It would be interesting to know the authors' views on the probable effect of the ground constants near the transmitter and the receiver on sky-wave propagation. Presumably where a long hop is involved, the signals leave the transmitter and arrive at the receiver at relatively low angles, and it seems that the properties of the ground near the terminal points must affect substantially the overall attenuation. In view of the notable success of the ground-wave treatment, perhaps the authors could, in due course, be persuaded to consider quantitatively the sky-wave case.

Mr. P. G. Redgment: The study of ground-wave propagation is of considerable interest in connection with radio navigational aids; the theory described in this paper has an obvious application to the estimation of field strength, and Loran—which operates on the ground wave at fairly high frequencies—is a typical case in which it should prove useful.

Some navigational aids operate by comparing phases of ground waves, and it is important to know what disturbance of phase takes place when the wave crosses a land/sea boundary. Experience indicates that there is a disturbance, but the results are too complicated to permit analysis of the effects on a given path. The authors' results, showing a large amplitude variation, seem to suggest considerable phase variation also. In the absence of a full theoretical solution it would be of great interest—and of very considerable practical importance—were we able to get a similar semi-empirical method for estimating the approximate magnitude of the phase change.

I think that this might be approached by an extension of the authors' work on attenuation. Unfortunately we are not concerned with a minimum-phase-shift network, and the usual relationship between phase and amplitude cannot be applied directly. However, it seems that the disturbance of amplitude and phase at the boundary could, perhaps, be regarded as perturbations due to a fictitious minimum-phase-shift network superimposed on the distributed network corresponding to normal propagation, and thus the phase shift could be calculated from the results given in the paper. I suggest that this may be well worth investigation.

It would be desirable to check any results thus obtained, and I should like to suggest two methods for consideration. For the case of a simple circular island with a transmitter in its centre, it might be possible to construct a network analogue of the field. Suitable inductors and capacitors could form a space network, with resistors or short-circuits to represent the land or sea surface. This might enable the exact form of the field round the boundary to be explored, and thus throw more light on what is actually taking place, as well as providing a check on the approximate theory.

Another possible approach lies in the use of a model such as the authors have shown. The difficulty here is instrumental—in the measurement of phase—but a rough idea of what is happening could be obtained by providing a standard of phase by a signal fed into the detector simultaneously with the radiated signal. A standing-wave pattern, appearing as an amplitude variation, will then be found; plotting this should enable the perturbations of phase at the boundary to be ascertained.

In describing the experiments, the authors mention the difficulty of field-strength measurement on shipboard. I am surprised that the loop was less satisfactory than the open wire, because in my experience a loop is less seriously affected than an open aerial by the various wire stays and other conducting objects found on board ship. I think that a more satisfactory calibration could have been made for a loop, though its effective height is changed by the presence of the ship's hull and must, of course, be determined empirically.

Mr. P. C. Clemmow: I am particularly interested in this work because recently I have been considering the problem from an analytical aspect. The case of a single boundary in a flat earth can be treated analytically, and numerical results obtained, although the process is lengthy. In this case, it appears that the authors' field-strength curve is very nearly correct; at the worst, when one of the media is a perfect conductor and the other a poor one, it gives field strengths which are only 2 db too great. If this agreement seems to be fortuitous, it does give excellent theoretical backing in addition to the good experimental justification already obtained.

It is implied in the paper that the 4-m experiment comes into the "short-wave-limit" category. But is this really so? The short-wave-limit case is discussed in Section 4 of Part 1 of the paper.* This Section demonstrates the geometric-mean formula given by equation (12). For a flat earth, the proof rests essentially on the fact that ray theory is applicable for the transmitter and receiver above a certain height, using an effective reflection coefficient of -1 which is independent of the nature of the ground. A necessary criterion for the applicability of ray theory is that the numerical distance must be large. In the 4-m experiment this condition is not satisfied. Over the distances concerned, the numerical distance for the sea is only of the order unity. It is worth mentioning that the geometric-mean formula is equally applicable to phase.

Referring again to Part 1 of the paper (*loc. cit.*), the argument for extending the graphical process beyond the short-wave limit seems to be confined to the diffraction region. I should like to know if the author has any ideas concerning an analogous approach for the longer wavelengths in the flat earth case—to which category the 4-m experiment belongs—in view of what I have said previously.

Mr. G. Bramslev (*Denmark: communicated*): The paper is of vital importance to the broadcast engineer wishing to determine the daylight range of medium-wave stations, for although the experimental investigations described have been carried out on much higher frequencies, it is probable that one of the greatest applications of the new method will be in connection with the practical use of the Sommerfeld curves in the medium-wave band, when the earth conductivity is changing materially with distance and direction from the transmitter.

The problem of computing ground-wave field strengths for

* HEISING, R. A.: "Effect of Shore Station Location Upon Signals," *Proceedings of the Institute of Radio Engineers*, 1932, 20, p. 77.

* *Proceedings I.E.E.*, 1949, 96, Part III, p. 53.

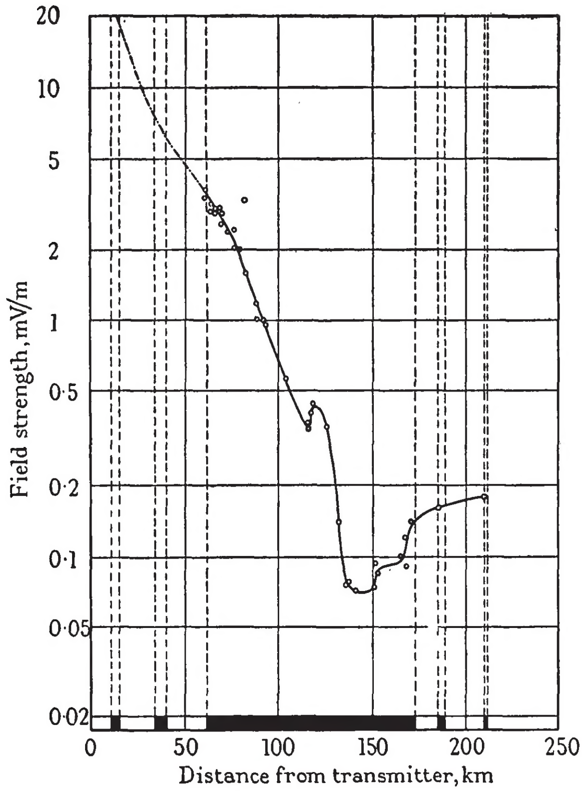

Fig. A.—Variation of field strength with distance.

Route: Kalundborg–Thyborøn.
Frequency: 1 060 kc/s.
Power: 1 kW.
Land paths.
Sea paths.

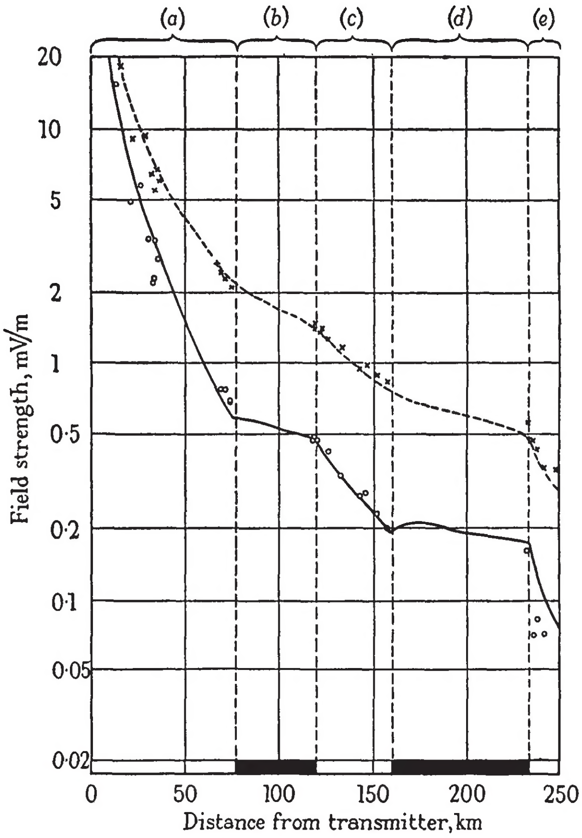

Fig. B.—Variation of field strength with distance.

Land paths. Sea paths.
o o o Measured values, 1 060 kc/s; —— Curves calculated by Millington's method, 1 060 kc/s.
× × × Measured values, 541 kc/s; - - - Curves calculated by Millington's method, 541 kc/s.

(a) $\sigma = 10 \times 10^{-14}$ e.m.u.
(b) Sea path.
(c) $\sigma = 10 \times 10^{-14}$ e.m.u.
(d) Sea path.
(e) $\sigma = 2 \times 10^{-14}$ e.m.u.

medium waves over composite paths has been a fairly important one for Denmark because of the geographical shape of the country, and it was, in fact, a particular case where an observed error in field-strength calculation up to about 14 db gave a false picture of the field-strength distribution, that led us to examine the problem experimentally. Since the publication of Part 1 of the paper, we have compared many of the experimentally determined curves with the Millington method, and have obtained such good agreement that we now employ it with full confidence over all mixed routes, even for the prediction of field strengths over large cities, although in this case the calculated values cannot be expected to give an entirely correct picture of the field distribution.

Figs. A and B are typical curves from our field-strength surveys over Denmark and Sweden. In Fig. A, where the operating frequency was 1 060 kc/s, the paths consisted of an initial stretch of 60 km of sea—including two small islands of minor importance—followed by 110 km of land and a further 40 km of sea—including one small peninsula. A very rapid drop in field strength is observed over one part of the land, followed by a recovery of about 6 db over the last part of this section. This recovery is continued out to sea and reaches a total of 8 db at the final point of the observed route. In this case we have not attempted to find agreement with the Millington method, as no means of determining the land conductivities was available. The curve is of interest because it shows that an exceptionally rapid drop followed by a recovery can exist on medium waves, not only as caused by the high attenuation offered to radio waves by large cities, but also where the bad conductivity can be attributed to moors, etc.

Another stretch, from Kalundborg across Seeland and Sweden to the island of Bornholm, was examined more carefully on 540 and 1 060 kc/s. It is an interesting example of the use of Millington's method along a route consisting of three land paths and two sea paths. The total length is 250 km, with the land paths of 76, 40 and 16 km respectively and the sea paths of 44 and 74 km. Careful measurements of field strength on both frequencies were made at a total of 30 points, including those near the transmitter used for determining the correct value of the radiated power. The curve for the first land path showed this to have a conductivity of 10^{-13} e.m.u. For the next land stretch, the radiation under test could not be used for the calculation of earth conductivity, but the Swedish broadcasting station at Malmö covered this area and was kindly placed at our disposal by the Swedish Telegraph Administration, and the conductivity was found to be also 10^{-13} e.m.u. for this part of the route. For the island of Bornholm, it has only been possible to estimate the conductivity to be 2×10^{-14} e.m.u. from an examination of geological maps, but lack of precision in this value will only affect the shape of the field curve over the last 10 or 15 km.

The results of the measurements are shown in Fig. B, together with the theoretical curves given by Millington's method. It will be seen that the agreement is very good indeed, in fact better than that obtainable in many field-strength surveys over apparently homogeneous ground. These curves, and others showing nearly as good agreement, have been taken by the

Danish Post and Telegraphs as proof of the practical application of the method on medium waves. There is, however, one point where we feel a little uncertain about employing Millington's method over mixed paths, namely the case of very narrow stretches of land or sea, where the conductivity changes considerably on both sides of a radial. As also pointed out by the authors, this case needs further investigation, both theoretical and practical.

I feel that we have now got a very practical and efficient engineering tool for use in the prediction of ground-wave field-strengths.

Mr. W. Struszynski: I should like to emphasize the importance of the authors' experimental discovery of "the recovery in field strength" effect. The results obtained provide an invaluable quantitative check on any theoretical formulae and therefore on the validity of the assumptions and approximations involved.

Referring to the problem raised by Dr. Smith-Rose, namely that of the distortion of the wavefront, I should like to comment on the physical side of it. The wavefront assumes a certain tilt at the earth's surface, instead of being vertical. This distortion results from a constant flow of electromagnetic energy from the upper region of the wave to the lower, in order to supply energy to the wave transmitted into the earth, the flow being due to the presence of an electromagnetic field at the interface of the two media. The energy transmitted into the earth directly from the aerial is rapidly attenuated by the poorly conducting earth. Therefore the required energy can be supplied only indirectly from air close to the surface. Normals to the wavefront indicate the direction of the flow of energy in that region, and the tilt of the wavefront, therefore, shows that this flow is not horizontal at the surface (see Fig. C). At the earth's surface, some phase lag occurs, which results in a slight reduction of the phase velocity of the wave in comparison with that in free space. This is explained by the fact that the energy is drawn from higher regions, so that distortion of the wavefront gradually spreads from the lower region to the higher (see Fig. D). The phase lag at the earth's surface increases with distance, but the rate of this increase becomes smaller, and in fact the ultimate phase lag for a flat earth would tend to π (i.e. $\lambda/2$). The corresponding phase velocity would gradually increase until it approached the velocity of light for very large distances. The tilt of the wavefront changes with the height, becoming gradually vertical.

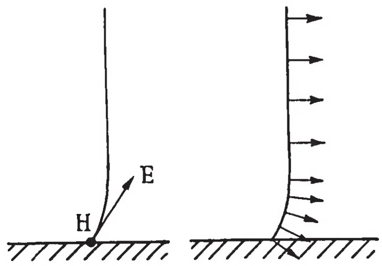

Fig. C

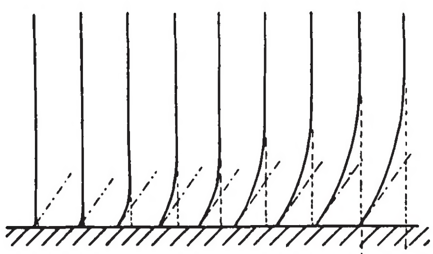

Fig. D

When the wavefront crosses a coast-line from land to sea, it has a large tilt corresponding to land conditions, and the downward flow of energy is still the same as it was over land. Since sea is a medium of much greater conductivity and dielectric constant than the land, less energy is transmitted into the sea, and so a considerable portion is reflected back to produce an increase of field intensity at the surface, thus explaining the effect of recovery in field intensity. At a great distance from the boundary, the conditions for transmission over the sea are established, with the corresponding tilt, since the downward flow of energy replenishes, at a sufficiently large distance, the loss in the intervening land path (see Fig. E).

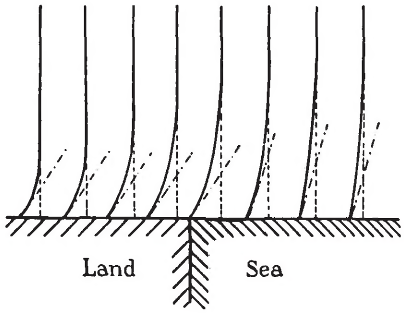

Fig. E

It is reasonable to expect a rapid advance of phase to accompany the field-recovery effect; this would correspond to a local increase of the phase velocity.

Mr. S. B. Smith (*communicated*): In Part 1 of the paper, Mr. Millington mentions that the method is not applicable to an approximate solution of the phase-surface conditions which are experienced when a wave is travelling obliquely across a mixed land/sea path. Usually, an amplitude variation of this type is accompanied by a phase variation. It is realized that these boundary conditions do not lend themselves to any simple evaluation of the constant phase-surface contours. Has the author any simple method which could be applied to these phase-surface calculations and also to any phase-recovery effects which must occur at a distance from the boundary?

In Part 2 of the paper, a brief reference is made to sky/ground-ray d.f. interference errors. Some time ago I was interested in a study where a marked sea recovery was present. Under these conditions, the effects upon the night-time accuracy of a Consol directional beacon were most apparent. If one considers a Consol beacon covering a uniformly conducting and smooth earth, or, for that matter, over the sea, the night-time standard deviation usually reaches a single maximum at a distance where the ground- and sky-wave field-intensities are equal in amplitude; this is due to some lateral deviations present in either or both rays. In a mixed-path case, where considerable recoveries are present, two well-defined standard-deviation maxima are to be expected, namely when the quasi-mean night field-intensities are equal to those of the ground rays at the appropriate distances. It would be interesting to know if such double-peaked standard-deviation curves have been observed by those using Consol or other beacons operating along suitable paths? Have the authors any data?

THE AUTHORS' REPLIES TO THE ABOVE DISCUSSION

Messrs. G. Millington and **G. A. Isted** (*in reply*): In answer to Dr. Smith-Rose, about the land/sea discontinuity in the 4-m experiment, it would probably be incorrect to allow for it in the height gain over the sea. It is the actual height of the receiving aerial above the sea that matters, and the difference in level at the boundary would produce a local disturbance which would modify the interference effects that may exist already. His question about the change in the tilt of the wave has been amplified by Mr. Struszynski. Undoubtedly the increase in field strength over the sea is associated with a change in direction of the energy flow above the earth. For a complete explanation it may be necessary to study the Poynting vector, though in a composite wave system, such as exists near the surface of the earth, the vertical electric field by itself may not suffice to define the energy density.

Our work cannot help Mr. Kirke with his problem of v.h.f. propagation over uneven country, as it is expressly limited to changes in the earth constants. Although there may be some connection between the diffraction associated with irregular terrain and that caused by inhomogeneities, we do not see how to apply our results directly to his problem. His point about the deduction of the difference in height gain on the two sides of the boundary from the recovery effect is surely only a reversal of our explanation of the latter by an appeal to the different height-gain functions over land and sea.

While we agree with Mr. Rowden that the Somerville method often gives a good approximation on medium waves, the remarks of Mr. Bramslev show that the recovery effect can be an important feature of medium-wave broadcasting. On 300 m, the contrast between the attenuation over land and sea, which is not great for a land conductivity of $\sigma = 10^{-13}$ e.m.u., increases rapidly when σ decreases. Thus for $\sigma = 10^{-14}$ e.m.u. the recovery can be comparable with that obtained on 100 m with $\sigma = 10^{-13}$ e.m.u. It may be noted that Mr. Elson* has found a marked recovery, in an aircraft flying at 1 000 ft across the Norfolk coast, from a transmitter operating on a wavelength of about 300 m at Crowborough.

We realize, with Mr. Bramslev, that the case of transmission along a boundary needs further study, although the experimental evidence suggests that waves travelling along a river only a few wavelengths wide are markedly affected by it. In fact, within wide limits, each radial can be treated on its own merits and the field-strength contours rounded off where they cross the boundaries.

Mr. Stanesby's question about the effect of the ground constants on low-angle sky-wave propagation is difficult to answer where the methods of geometrical optics do not apply. We feel, however, that it is somewhat off the theme of our paper, as it refers to the effect of the earth constants as such and not specifically to their inhomogeneity.

With regard to Mr. Redgment's query about the aerial used on board ship, it must be remembered that the measurements had to be made without any preliminary trial on a ship that was on a normal run with a complement of passengers. It was found that the loop aerial would not function properly under the given conditions, but that, fortunately, the existing vertical aerial was quite satisfactory for relative measurements.

We have no data to offer Mr. Smith in support of double-peaked standard-deviation curves resulting from the recovery effect, though it is an interesting possibility and one that may well give some indirect confirmation of our theory. On the wavelengths he has in mind, however, the phenomenon should be that of a hold-up rather than of a recovery in field strength, and we doubt whether the standard-deviation/distance curves would show more than a single broadened peak.

Mr. Smith and Mr. Redgment both raise the vital question of the phase changes across a land/sea boundary in connection with coastal refraction and phase comparison methods of navigation. It is here that the work described by Mr. Clemmow* holds great possibilities. We are indebted to him for pointing out that where our theory gives the amplitude of the wave well beyond the boundary it also implicitly gives the phase. We regard his work as of fundamental importance, as it gives our results the rigid mathematical support they need in the immediate neighbourhood of the boundary. It is indeed gratifying that our empirical argument in this region is so strikingly confirmed, and we can only claim that if we made a guess it was at least an intelligent one.

Admittedly our height-gain argument does not apply in the case of a perfect conductor, but the formal application of our method still leads to the correct rate of attenuation far enough away from the boundary over the conductor. It therefore seemed reasonable to apply the same empirical argument at the boundary, and to suppose that the final level of the curve is not greatly in error. We feel that it is a happy circumstance that our respective methods go so closely hand in hand. It is doubtful whether it would have been appreciated that the results of the rigid analysis can be so nearly represented by such a simple graphical process had we not arrived at the latter by an independent argument.

On the other hand, the rigid treatment is invaluable in assuring the engineer that the convenient technique provided by our method is, for all practical purposes, correct and may be applied with confidence when there is more than one boundary and where the curvature of the earth has to be taken into account. We are encouraged to hope that our method may in due course be extended to apply to the phase relationships and that it may similarly give an engineering solution that will be confirmed by the rigid theory.

Meanwhile, by again invoking the height-gain argument and considering the part of the radiated wave well above the ground, it appears that, at a sufficient distance beyond the boundary, the wave on the ground will be travelling parallel to its original direction. The distance over which a rapid phase change will take place will probably be about the same distance as the maximum of the recovery effect beyond the boundary. A map such as we have shown of the amplitude contours round an island will thus give a good indication of the region of phase disturbance.

* ELSON, N.: "Ground-Wave Propagation across a Land/Sea Boundary; 300-m Waves," *Nature*, 1949, 164, p. 114.

* CLEMMOW, P. C.: "Ground-Wave Propagation across a Land/Sea Boundary," *Nature*, 1950, 165, p. 107.